DISTRIBUTION OF THE PHASE FRONT IN THE FREEZING OF FINELY-DISPERSED SOILS

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The process of the freezing of soils is examined with allowance for the migration of moisture in the freezing and thaw zones.

The laws governing the transfer of heat and moisture in finely dispersed soils have been the subject of a large number of theoretical studies. However, these studies have employed highly simplified models to describe complex phenomena. Without discussing different approaches to the description of the freezing of finely dispersed soils (see [1, 2], etc.), we note only that most previous investigations did not consider the kinetics of the phase transformation and the migration of moisture in the frozen zone. Thus, characteristic features were excluded from examination, particularly those phenomena which govern stratification. The authors of [3] proposed a physically substantiated model which considers the specifics of phase transformation in soil (the presence of unfrozen water) and heat and mass transfer in the thaw and freezing zones. The goal of the present investigation is to use this model as a basis for studying laws governing the motion of the freezing front in finely dispersed soils. We also want to study the thermal and moisture structures in the transformation zone.

In accordance with [3], we write the system of equations describing heat and moisture transfer in the form

$$\frac{\partial f_k}{\partial \tau} = \frac{\partial}{\partial x}\left(\gamma_k \frac{\partial f_k}{\partial x}\right) + e_k \frac{\partial L}{\partial \tau}, \quad k = 1,\ 2, \tag{1}$$

$$\frac{\partial L}{\partial \tau} = g(t,\ W,\ L), \tag{2}$$

where $f_1 = t$; $f_2 = W$; $\gamma_1 = a$; $\gamma_2 = D$; $e_1 = \kappa/c$; $e_2 = -1$.

The function g(t, W, L), determining the rate of crystallization of the water in the case of unidirectional freezing, can be represented in the form [4]

$$g = (W - W_e)/\tau_*, \tag{3}$$

where $W_e = W_e(t)$ is the amount of unfrozen water (determined on the basis of experimental data on the freezing of soil specimens). It should be noted that allowing for migrational phenomena in the freezing zone and thus considering the disequilibrium of the ice formation process in the case $t < t_e^0$ may result in a system characterized by a state in which $W - W_e < 0$. In contrast to melting processes ($L > 0$), this circumstance presumes the existence of a region in which no phase transformations take place, i.e., the rate of transformation becomes equal to zero. Allowance is made for this in the computing process when developing the procedure for calculating the kinetic function g(t, W, L).

Henceforth restricting ourselves to the case of the freezing of finite specimens with boundaries impermeable to moisture (closed system), we write the initial and boundary conditions corresponding to the problem being examined:

$$\tau = 0, \quad 0 \leqslant x \leqslant H \quad t = t_0, \quad W = W_0, \tag{4}$$

$$\tau > 0 \begin{cases} x = 0 \quad t = t_1, \quad \dfrac{\partial W}{\partial x} = 0, \\ x = H \quad t = t_2, \quad \dfrac{\partial W}{\partial x} = 0, \end{cases} \tag{5}$$

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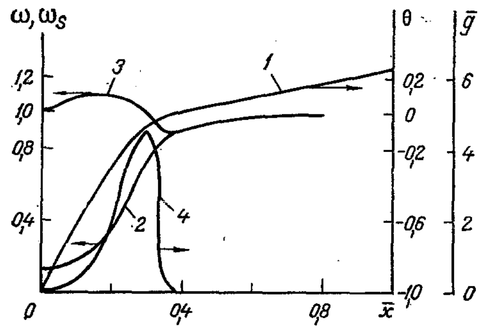

Fig. 1

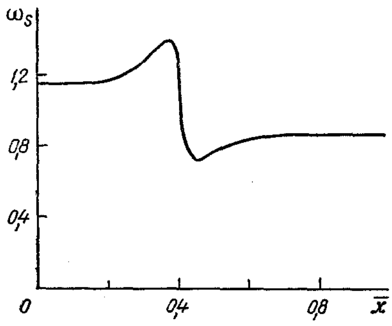

Fig. 2

Fig. 1. Distribution of temperature (1), moisture (2), total moisture content (3), and crystallization rate (4) in the specimen, Fo = 0.2; Le = 0.3; Ste = 0.23; θ_2 = 0.25.

Fig. 2. Measurement of total moisture content along the specimen, Fo = 1.8; Le = 0.3; Ste = 0.114; θ_2 = 0.875.

where the subscripts 0, 1, and 2 correspond, respectively, to the initial state and parameters on the "upper" and "lower" boundaries of the specimen ($t_1 < t_2 = t_0$).

In the numerical study of propagation of the phase front, Eqs. (1) and (2), initial conditions (4), and boundary conditions (5) were reduced by means of the transformation $f_1 = \theta = (t - t_e{}^0)/|t_1 - t_e{}^0|$, $f_2 = \omega = W/W_0$, $\omega_e = W_eW_0$, $\ell = L/W_0$, $\varphi = D(W)/D_0$, $\bar{x} = x/H$ (H is the specimen length) to dimensionless form:

$$\frac{\partial\theta}{\partial \mathrm{Fo}} = \frac{\partial^2\theta}{\partial\bar{x}^2} + \frac{1}{\mathrm{Ste}}\bar{g}, \quad \frac{\partial\omega}{\partial \mathrm{Fo}} = \mathrm{Le}\frac{\partial}{\partial\bar{x}}\left[\varphi(\omega)\frac{\partial\omega}{\partial\bar{x}}\right] - \bar{g}, \quad \frac{\partial l}{\partial \mathrm{Fo}} = \bar{g}, \tag{6}$$

$$\mathrm{Fo} = 0, \quad 0 \leqslant \bar{x} \leqslant 1, \quad \theta = \theta_0, \quad \omega = 1, \tag{7}$$

$$\mathrm{Fo} > 0 \begin{cases} \bar{x} = 0 \quad \theta = -1, & \dfrac{\partial\omega}{\partial\bar{x}} = 0, \\ \bar{x} = 1 \quad \theta = \theta_2, & \dfrac{\partial\omega}{\partial\bar{x}} = 0, \end{cases} \tag{8}$$

where $\mathrm{Le} = D_0/a$; $\mathrm{Ste} = c|t_1 - t_e{}^0|/\rho W_0\kappa$; $\bar{g} = (\omega - \omega_e)/\mathrm{Fo}_*$; $D_0 = D(W_0)$; $t_e{}^0$ is the temperature at which the phase transformations begin.

System (6) must be augmented by the relation $\omega_e(\theta)$, which together with (3) determines the form of the kinetic function. An analysis of the empirical data on the freezing of specimens of finely dispersed soils in the temperature range $\Delta t = t_1 - t_e{}^0$ [3, 5] leads to the following expression for $\omega_e(\theta)$:

$$\omega_e(\theta) = (1 - n\theta)^{-1}, \tag{9}$$

where $n \approx 7.7$.

In numerically solving system (6), we used an implicit scheme of approximation involving the use of central differences for the space coordinates and one-sided differences for time.†

The transport coefficients γ_k were referred to half-integral nodes. Thus, the error of approximation was of the order $O(h_\tau + h^2)$, where h_τ and h are the time and space steps, respectively. Thus, the problem being examined is nonlinear, and in the solution of its finite difference analog we combined the trial run method with iterations. For faster convergence of the iteration process, the value of $\bar{g}(\omega, \theta, \ell)$ was corrected after we found each of the grid functions being determined. The boundary conditions for the moisture transport equation

†In the calculations we performed, we assumed that the heat capacity and diffusivity of the soils were constant and were the same for the freezing and thaw zones. The dimensionless diffusion coefficient was determined by the expression $\varphi = \omega^8$ [5].

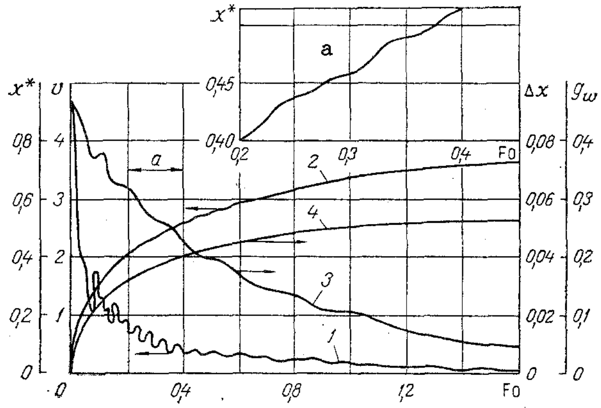

Fig. 3. Change over time in the rate of displacement (1) and position (2) of the thermal front, the migrative flow (3), and the region ($\theta < 0$) in which phase transformations are absent (4), Le = 0.3; Ste = 0.23; $\theta_2 = 0.25$.

were approximated by a central difference with the use of hypothetical nodes and subsequent expression of the grid function at these nodes from the difference analog of the differential equation. Such an approach results in time coupling at the boundary nodes, produces a uniform scheme, and, most importantly, increases the order of the approximation.

It is easily shown that by virtue of the finiteness of the functions $\bar{g}(\omega, \theta, \ell)$, following from the formulation of the problem of phase transitions at a finite rate, the chosen scheme results in a well-conditioned boundary-value problem and, thus, to a unique solution and stable calculation [6]. Along with the above considerations, special attention must be given in the calculations to the independence (in the numerical sense) of the solution from the subdivision of the grid region. The fields in the zone of intensive phase transformations were analyzed most intensively, since the largest gradients of the sought functions were to be found here.

In order to make a detailed description of kinetic processes and analyze their effect on the formation of the structure of the front, the time step was made smaller than the characteristic time Fo_*. The value of h_τ itself was varied repeatedly. We approached the space step h in a similar manner. It is very important the time step chosen for the given space discretization does not lead to changes (at the same moments of time) in the velocity of the front and leaves it a finite quantity. We only want to obtain a more detailed description of the oscillatory process. Here, the frequency and amplitude of the velocity oscillations remained unchanged. It follows from the second equation of system (6) that the total moisture content $I_0=\int_0^1 \omega_s d\bar{x}$ remains the same in a closed system. In dimensionless variables, $I_0 = 1$ and is the initial moisture content. This quantity was checked during the computation. The error connected with its maximum deviation from unity was no greater than $\varepsilon = 6\cdot 10^{-3}$. The analysis also showed that the difference scheme is absolutely stable with a change in the Lewis number in the range $0 < Le < \infty$. However, it should be noted that the moisture transport equation becomes rigid at Le = 0. This places certain restrictions on the size of the time step in order to satisfy static and dynamic stability conditions [7].

Figure 1 shows the main characteristics of the crystallization process in a specimen at a fixed moment of time. We should point out the presence of the distinct zone of intensive phase transformations δ. Outside this zone, the crystallization rate is negligibly small. At the given values of the parameters (Le ~ 0.3-1.0, Ste = 0.23), the width of the transformation zone δ is on the order of 0.1 H. This makes it impossible to represent the latter in the form of an infinitely thin freezing front.

The calculations show that the temperature distribution is nearly linear outside the transformation region in the freezing and thaw zones. This approach to linearity is evidence of the quasisteady nature of the heat transfer process. The moisture content distribution is quite nonlinear in character and is described by an S-shaped curve having zero derivatives at the boundaries. It is evident from Fig. 1 that, with increasing distance from the cold end, ω increases monotonically to the value corresponding to the moisture content of the soil in the thaw zone. The region in which ω changes sharply corresponds to the transformation zone, which directs the migration of moisture from the thaw zone due to crystallization and the corresponding increase in the potential ω. This in turn leads to an increase in the total moisture content $\omega_s = \omega + \ell$ in the frozen zone.

The distribution of total moisture content is very complex in character — it initially increases with increasing distance from the cold boundary ω_s and then decreases to the value corresponding to the equilibrium moisture content. A solid phase is absent in the range of $\bar{x}$ from $\bar{x}^*$ to $\bar{x}^{**}$ ($\bar{x}^*$ and $\bar{x}^{**}$ are the coordinates of the point corresponding to the beginning of the freezing of water in the soil and the point at which the condition $\omega_s = \omega_e$) is satisfied. The lack of solid phase can be attributed to the reduction in moisture content to values below equilibrium due to the migration of moisture into the frozen zone.

The character of distribution of temperature in the frozen and thaw zones and the moisture and ice contents are also maintained at other values of the parameters. A change in the latter is accompanied by restructuring of the fields of the respective quantities. In particular, an increase in Le leads to a slight contraction of the transformation zone, shifting of the crystallization-rate maximum toward the cold boundary, and expansion of the region in which phase transformations are absent at temperatures below the freezing point. Intensification of moisture transport (an increase in Le) leads to an appreciable displacement of the high-ice-content region toward the cold boundary. This is evidence of possible separation of the zone of active ice formation from the freezing front and the formation of a stratified texture near the cold boundary. Such a phenomenon, characteristic of both closed and open systems, has been observed in tests involving the freezing of specimens [2].

The distribution of total moisture content is significantly affected by the temperature head $\Delta = t_2 - t_1$ and the initial moisture content W_0. An increase in Δ, as a reduction in Ste, leads to localization of intensive ice formation in a narrow region near the freezing front (Fig. 2).

Figure 3 shows data on the velocity of the thermal front, its displacement over time, the migrative flow, and the difference Δx. The latter characterizes the size of the region in which phase transitions are absent at negative temperatures. We should point out the nonmonotonic character of the change in the velocity of the front and the migrative flow. In accordance with this, there is also a change in the function $\Delta\bar{x}^*(\text{Fo})$ (see fragment a). The oscillations of the quantities v and q_W are evidently due to inequality of the rates of moisture and heat transfer owing to the difference in the relations $a(W)$ and $D(W)$. In fact, when the front is displaced from the cold boundary, there is a substantial reduction in moisture content near this boundary due to crystallization of water in the transformation zone. This in turn leads to a sharp reduction in the diffusion coefficient ($D \sim W^8$) and, accordingly, to a reduction in the migrative flow from the thaw zone. With a constant heat flux (diffusivity is a weak function of W and we assume that $a = \text{const}$), a decrease in q_W leads to an increase in the velocity of the front in the region of high moisture contents, where velocity again decreases. An increase in the ratio D_0/a helps generate oscillations, since the reduction in moisture content caused by crystallization is partially or completely offset by the flow of moisture from the thaw zone.

The final stage of the freezing process corresponds to stoppage of the front a certain distance from the cold boundary. This distance depends on the temperature head and the thermophysical and migrative characteristics of the soil. Calculations show (curve 3, Fig. 3) that when the system attains the steady state, migration of moisture from the thaw zone nearly ceases. This is also evidence by the fact that the length of the zone $\Delta\bar{x}$ tends toward a finite limit as $\text{Fo} \to \infty$. This is the zone in which phase transitions are absent in the negative temperature region. These findings are consistent with the experimental data in [8].

NOTATION

τ, x, time and space coordinates; t, W, L, dimensionless values of temperature, moisture content, and ice content; c, a, D, volumetric heat capacity, diffusivity, and diffusion of moisture; ρ, density of the skeleton; W_e, equilibrium value of moisture content; κ, enthalpy of phase transformations; τ_*, characteristic time; θ, ω, ℓ, φ, dimensionless values of temperature, moisture content, ice content, and diffusion coefficient of the moisture; Fo, Fourier criterion; Ste, Stefan number; n, empirical constant. The indices 0, 1, and 2 pertain to the initial and boundary states.

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CONVERSE THERMAL CONDUCTIVITY PROBLEMS AND CALORIMETRY OF TRANSPARENT BODIES

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The problem of measuring temperature of transparent bodies can be solved by use of transparent thin film resistance thermometers. Such sensors have been developed using tin and indium oxide. They are used to perform calorimetry of the properties of partially transparent bodies and laser radiation.

Colorimetry consists of methods for measuring the thermal effects accompanying physical, chemical, and biological processes. For the present we will understand this term to mean methods for determining the thermophysical properties of transparent bodies,* together with methods for determining energy characteristics of radiation.

The characteristics of laser radiation are most often determined by calorimeters with a load (calorimetric body) consisting of a more or less perfect ideal black body model [1]. In some cases it is desirable to use a completely or partially transparent colorimetric body, although problems then develop in measuring its temperature. If the temperature sensor has thermophysical and optical characteristics differing from those of the body whose temperature is to be measured, then the presence of other surrounding bodies with different temperatures, or the presence of radiation, either one cannot in principle measure the temperature of the given transparent body, or that measurement will require introduction of corrections which often are of significant complexity [2]. The difficulties in temperature measurement increase when the energy transport process is of a transient nature. In this case the corrections to the measurement may exceed the level of the temperature itself and change their algebraic sign in various stages of the process [3]. Thus, in recent years there has been a deliberate search for methods of measuring the temperature of transparent bodies. It is obvious that for this purpose one may use any phenomenon in which any optical or electrical characteristic of a substance changes with temperature [4-7].

One possibility for measuring temperature of transparent bodies reduces to creation and use of conductive thin-film coatings of a material transparent to the given kind of radiation, which are deposited on the surface of the body. Such thin conductive films can be used as

*The concept of a "transparent" body is an idealization. Herein by transparent we will understand bodies in which absorption of some portion of the passing radiation does occur.

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